R E V I S E D 3/11/79

THE MIRROR-ROTATION SYMMETRY

The task:

To make a mirror-rotation object that demonstrates the principle of this symmetry; to refine it aesthetically; and to execute it from a block of wood or in other appropriate material.

The principle:

If upon a figure, 2-dimensional or 3-dimensional, any one or several of various geometric operations can be performed, such that there is no apparent change, even though there is in fact an actual transformation, that figure may be said to have symmetric quality(ies).

There are six isometric coverage operations: three primitive operations -- translation, mirror, rotation; three coupled operations -- translation-mirror (or glide), translation-rotation (or screw or helical), and mirror-rotation. The last of these six isometries is little known, little employed, and not easily recognized. Analogously, many people do not recognize that face cards of the conventional playing-card deck are patterned to a two-fold rotation rather than to a bilateral symmetry. It is even more difficult to recognize objects of mirror-rotation symmetry -- we are so overwhelmingly familiar with the bilateral symmetry that our own bodies display.

This symmetry is, however, known to chemists, who recognize it in certain molecular structures, and to crystallographers, who find it sometimes as the only symmetrical quality possessed by certain crystalline structures -- both of whom call it "centered" symmetry.

While puzzling to the senses and, therefore, intriguing to the mind, it is remarkably easily described and demonstrated. Any two 3-dimensional objects that are congruent of shape and size but opposite handed (more properly, have right and left screw senses) may be formed into this symmetry by rotating them 180^{o}s from a mirror reflection conformation. This can easily be demonstrated with one's two hands, merely by juxtaposing the hands at a 180^{o} turn. And any mirror-rotation figure, which is cut, at any direction, straight through its center (of gravity), will produce two equal but oppositely handed parts. It exists in all regular figures: cube, sphere, octahedron, etc. -- but rather hidden in these cases, since less complicated, i.e., more readily comprehended symmetries dominate one's perception.

It can also be described as an inversion, in three-dimensions, through a point. The camera, with its lens at the focal point, works on this principle. (An inversion through a point in the plane creates a 2-fold playing-card symmetry. An inversion across a line or plane produces the familiar mirror symmetry in the two- or three-dimensional situation.)

The pedagogic goal:

To have the student become totally familiar with the unique properties of this most recently identified symmetry and to have him fashion an object according to it that is aesthetically satisfying in proportion, contour, shape, configuration. Solutions tend to be either (1) minimally simple or (2) didactic or (3) of baroque complexity, yet with harmonic cohesion.

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